Subject Code:24BM11RC05 R-24 Reg No: | | | | | | | | | | | |

GAYATRI VIDYA PARISHAD COLLEGE OF ENGINEERING FOR WOMEN
(Autonomous)
(Affiliated to Andhra University, Visakhapatnam)
IIB.Tech. - I Semester Regular Examinations, Nov – 2025
DISCRETE MATHEMATICAL STRUCTURES
(Common to CSE, CSE (AI&ML), IT)

1. All questions carry equal marks
2. Must answer all parts of the question at one place

Time: 3Hrs. **Max Marks: 70**

UNIT-I

1. a. Show that the relation R = {(x, y) / x-y is an integer} is an equivalence relation. (7M)

   b. LetA be a given finite set and P (A) its power set. Let $\subseteq$ be the inclusion relation on the elements of P (A). Draw Hasse diagrams of (P (A), $\subseteq$) for A= {a,b,c} (7M)

OR

2. a. (i). Describe the transitive closure of a relation with an example? (3M)

   (ii). Explain about the types of functions in detail. (4M)

   b. Determine the number of positive integers n such that $1 \leq n \leq 100$ and n is notdivisible by 2,3 or 5. (7M)

UNIT-II

3. a.(i). Show that $P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$ (3M)

   (ii). Obtain the principal disjunctive normal forms of $(P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$(4M)

   b. Show the following premises are inconsistent. (7M)

   If Jack misses many classes through illness, then he fails high school.

   If Jack fails high school, then he is uneducated.

   If Jack reads lot of books, then he is not uneducated.

   Jack misses many classes through illness and reads lot of books'

OR

4. a. Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$, $\sim R \vee P$, and Q. (7M)

   b. Symbolize the following argument and check for its validity: (7M)

   Every living thing is a plant or an animal

   David's dog is alive and it is not a plant

   All animals have hearts

   Therefore, David's dog has an heart

UNIT-III

5. a. Prove by mathematical induction that for all integers $n \geq 1$: (7M)

$$1+3+5+\cdots+(2n-1)=n^2$$

b. Use the Euclidean Algorithm to find gcd of (414,662). (7M)

OR

6. a. Use the Chinese Remainder Theorem to solve the system of congruences: (7M)

$$x \equiv 2 \pmod 3, \quad x \equiv 3 \pmod 4, \quad x \equiv 2 \pmod 5.$$

b. State Fermat's Little Theorem and use it to find the remainder when 7^{222} is divided by 13.(7M)

UNIT-IV

7. a. Solve recurrence relation $a_n=3a_{n-1}+2a_{n-2}$ for $n>=2, a_0=1, a1=2$ using characteristic roots? (7M)

b. Solve the recurrence relation $a_n = a_{n-1} + n$ where $a0 = 2$, by substitution method. (7M)

OR

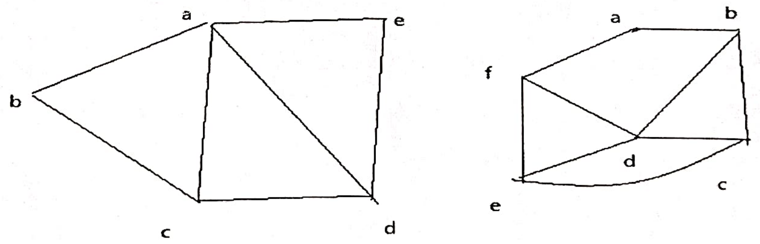
8. a. Solve the recurrence relation $a_n - 7a_{n-1} + 10\,a_{n-2} = 0$ where $n \geq 2$, by generating functions. (10M)

b. A coin is tossed 10 times. How many possible outcomes are there? (4M)

UNIT-V

9. a. What is Graph Isomorphism? Discuss the Isomorphism of graphs with an example? (7M)

b. (i) what is chromatic number of a graph? Findthe chromatic number of the following graphs. (3M)

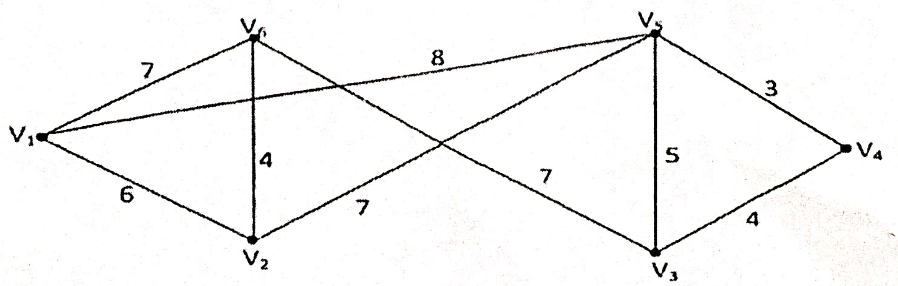


(ii). Elaborate the Euler and Hamiltonian graphs? (4M)

OR

10. a. Explain the graph traversals (BFS and DFS) with an example? (7M)

b. Using Prim's algorithm, to find a minimal spanning tree for the following weighted graph. (7M)



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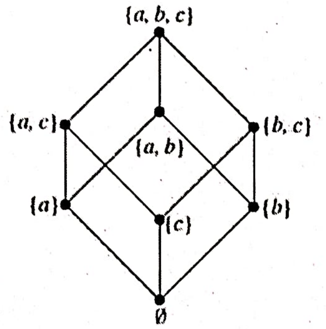
Key

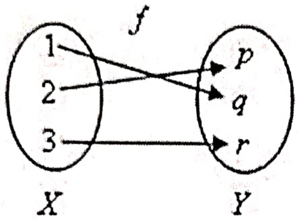
1a) show that {(x,y)/ x-y is an integer) is an equivalence relation
Solution:
For reflexivity, $x-x=0$ is an integer
For symmetry, if x-y is an integer then y-x is also an integer
For transitivity, if x-y is an integer and y-z is are integers then x-z=(x-y)-(y-z) is also an integer
Therefore, it is an equivalence relation because it is reflexive, symmetric, and transitive
1b) Let A be a given finite set and P(A) its power set. Let $\subseteq$ be the inclusion relation on the elements of P(A). Draw the Hasse diagram of (P(A),$\subseteq$)for A={a,b,c}.
Solution:
$A=\{a,b,c\}$
P(A)={∅,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}}



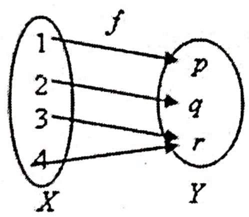
2a) (i) Describe the transitive closure of a relation with example
Solution:
Let X be any finite set and R be a relation in X.
The relation $R^+ = R \cup R^2 \cup R^3 \cup \cdots \cup R^n$ in X is called the *transitive closure* of R in X.
Example: Let the relation $R = \{(1, 2), (2, 3), (3, 3)\}$ on the set $\{1, 2, 3\}$.
$R = \{(1, 2), (2, 3), (3, 3)\}$
$R^2 = R \circ R = \{(1, 2), (2, 3), (3, 3)\} \circ \{(1, 2), (2, 3), (3, 3)\} = \{(1, 3), (2, 3), (3, 3)\}$
$R^3 = R^2 \circ R = \{(1, 3), (2, 3), (3, 3)\}$
$R^4 = R^3 \circ R = \{(1, 3), (2, 3), (3, 3)\}$
$R^+ = R \cup R^2 \cup R^3 \cup \cdots \cup R^n$
$= \{(1, 2), (2, 3), (3, 3)\} \cup \{(1, 3), (2, 3), (3, 3)\} \cup \{(1, 3), (2, 3), (3, 3)\} \cup \ldots$
$= \{(1, 2), (1, 3), (2, 3), (3, 3)\}$.
Therefore $R+ = \{(1, 2), (1, 3), (2, 3), (3, 3)\}$.

2a) (ii) Explain about the types of functions in detail

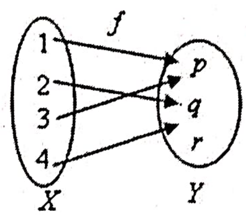
Soluton:
One-to-one(Injection): A mapping $f : X \rightarrow Y$ is called *one-to-one* if distinct elements of X are mapped into distinct elements of Y, i.e., f is one-to-one if
$x1 \neq x2 \Rightarrow f(x1) \neq f(x2)$
or equivalently $f(x1) = f(x2) \Rightarrow x1 = x2$ for $x1, x2 \in X$.



Example: $f : R \rightarrow R$ defined by $f(x) = 3x$, $\forall x \in R$ is one-one, since
$f(x1) = f(x2) \Rightarrow 3x1 = 3x2 \Rightarrow x1 = x2$, $\forall x1, x2 \in R$.
Onto(Surjection): A mapping $f : X \rightarrow Y$ is called *onto* if the range set $Rf = Y$.
If $f : X \rightarrow Y$ is onto, then each element of Y is f-image of atleast one element of X.
i.e., $\{f(x) : x \in X\} = Y$.
If f is not onto, then it is said to be *into*.

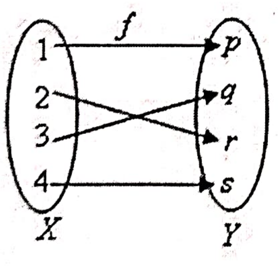


Surjective



Not Surjective

Example: $f : R \rightarrow R$, given by $f(x) = 2x$, $\forall x \in R$ is onto.
Bijection or One-to-One, Onto: A mapping $f : X \rightarrow Y$ is called *one-to-one, onto* or *bijective* if it is both one-to-one and onto. Such a mapping is also called a one-to-one correspondence between X and Y.



Identity function: Let X be any set and f be a function such that $f : X \rightarrow X$ is defined by $f(x) = x$ for all $x \in X$. Then, f is called the identity function or identity transformation on X. It can be denoted by I or Ix.
Note: The identity function is both one-to-one and onto.
Let $Ix(x) = Ix(y)$
$\Rightarrow x = y$
$\Rightarrow Ix$ is one-to-one
Ix is onto since $x = Ix(x)$ for all x.

2b) Determine the number of positive integers n such that 1≤n≤100 and n is not divisible by 2,3 or 5

Solution:

Number of integers divisible by 2: $N(2) = \lfloor \frac{100}{2} \rfloor = 50$

Number of integers divisible by 3: $N(3) = \lfloor \frac{100}{3} \rfloor = 33$

Number of integers divisible by 5: $N(5) = \lfloor \frac{100}{5} \rfloor = 20$

Number of integers divisible by 2 and 3 (i.e., by 6): $N(6) = \lfloor \frac{100}{6} \rfloor = 16$

Number of integers divisible by 2 and 5 (i.e., by 10): $N(10) = \lfloor \frac{100}{10} \rfloor = 10$

Number of integers divisible by 3 and 5 (i.e., by 15): $N(15) = \lfloor \frac{100}{15} \rfloor = 6$

Number of integers divisible by 2, 3, and 5 (i.e., by 30): $N(30) = \lfloor \frac{100}{30} \rfloor = 3$

sing the Principle of Inclusion-Exclusion, the number of integers divisible by 2, 3, or 5

:

$$(N(2) + N(3) + N(5)) - (N(6) + N(10) + N(15)) + N(30)$$

$$(50 + 33 + 20) - (16 + 10 + 6) + 3$$

$$103 - 32 + 3 = 74$$

Subtract the result from the total number of integers.
Total integers - (Integers divisible by 2, 3, or 5) =100-74=26

3 a) i) Show that $P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$

Solution: $P \rightarrow (Q \rightarrow R) \Leftrightarrow P \rightarrow (\neg Q \vee R)$ [$\because Q \rightarrow R \Leftrightarrow \neg Q \vee R$]

$\Leftrightarrow \neg P \vee (\neg Q \vee R)$ [$\because P \rightarrow Q \Leftrightarrow \neg P \vee Q$]

$\Leftrightarrow (\neg P \vee \neg Q) \vee R$ [by Associative laws]

$\Leftrightarrow \neg(P \wedge Q) \vee R$ [by De Morgan's laws]

$\Leftrightarrow (P \wedge Q) \rightarrow R$[$\because P \rightarrow Q \Leftrightarrow \neg P \vee Q$].

3 a) ii) Obtain the PDNF for $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$.

Solution: Express each conjunction in terms of all literals:
$(P\wedge Q\wedge R)\vee(P\wedge Q\wedge\neg R)\vee(\neg P\wedge Q\wedge R)\vee(\neg P\wedge\neg Q\wedge R)\vee(P\wedge\neg Q\wedge R)\vee(\neg P\wedge Q\wedge\neg R)$

Combine all the terms to form the PDNF:
$(P\wedge Q\wedge R)\vee(P\wedge Q\wedge\neg R)\vee(\neg P\wedge Q\wedge R)\vee(\neg P\wedge\neg Q\wedge R)\vee(P\wedge\neg Q\wedge R)\vee(\neg P\wedge Q\wedge\neg R)$

Final Answer:
$(P\wedge Q\wedge R)\vee(P\wedge Q\wedge\neg R)\vee(\neg P\wedge Q\wedge R)\vee(\neg P\wedge\neg Q\wedge R)\vee(P\wedge\neg Q\wedge R)\vee(\neg P\wedge Q\wedge\neg R)$

3 b) Show that the following premises are inconsistent.
(i) If jack misses many classes through illness, then he fails high school
(ii) If jack fails high school, then he is uneducated.
(iii) If jack reads lot of books, then he is not uneducated.
(iv) Jack misses many classes through illness and lot of books.

Solution:

Let us indicate the statements as follows:

E: Jack misses many classes through illness.

S: Jack fails high school.

A: Jack reads a lot of books.

H: Jack is uneducated.

The premises are $E \to S$, $S \to H$, $A \to \neg H$, and $E \wedge A$.

{1}	(1)	$E \to S$	Rule P
{2}	(2)	$S \to H$	Rule P
{1, 2}	(3)	$E \to H$	Rule T, (1), (2), and I13
{4}	(4)	$A \to \neg H$	Rule P
{4}	(5)	$H \to \neg A$	Rule T, (4), and $P \to Q \Leftrightarrow \neg Q \to \neg P$
{1, 2, 4}	(6)	$E \to \neg A$	Rule T, (3), (5), and I13
{1, 2, 4}	(7)	$\neg E \vee \neg A$	Rule T, (6) and $P \to Q \Leftrightarrow \neg P \vee Q$
{1, 2, 4}	(8)	$\neg(E \wedge A)$	Rule T, (7), and $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$
{9}	(9)	$E \wedge A$	Rule P
{1, 2, 4, 9}	(10)	$\neg(E \wedge A) \wedge (E \wedge A)$	Rule T, (8), (9) and I9

Thus, the given set of premises leads to a contradiction and hence it is inconsistent

4 a) Show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\neg R \vee P$, and Q.

Solution: Instead of deriving $R \to S$, we shall include R as an additional premise and show S first.

{1}	(1)	$\neg R \vee P$	Rule P
{2}	(2)	R	Rule P (assumed premise)
{1, 2}	(3)	P	Rule T, (1), (2), and I10
{4}	(4)	$P \to (Q \to S)$	Rule P
{1, 2, 4}	(5)	$Q \to S$	Rule T, (3), (4), and I11
{6}	(6)	Q	Rule P
{1, 2, 4, 6}	(7)	S	Rule T, (5), (6), and I11
{1, 2, 4, 6}	(8)	$R \to S$	Rule CP

4 b) Symbolize the following argument and check for its validity
Every living thing is a plant or an animal. David's dog is alive and it is not a plant. All animals have hearts. Therefore, David's dog has a heart.

Solution:

Let $L(x)$: x is a living thing; $P(x)$: x is a plant ; $A(x)$: x is an animal; $H(x)$: x have heart

Let y represents David's dog

The inference is $\forall x\left(L(x) \to (P(x) \vee A(x))\right), L(y) \wedge \sim P(y), \forall x(A(x) \to H(x)) \Rightarrow H(y)$

1.	$\forall x\left(L(x) \to (P(x) \vee A(x))\right)$	Rule P
2.	$L(y) \wedge \sim P(y)$	Rule P
3.	$\forall x(A(x) \to H(x))$	Rule P
4.	$L(y) \to (P(y) \vee A(y))$	Rule T,1,US
5.	$A(y) \to H(y)$	Rule T,3,US
6.	$L(y)$	Rule T,2, $p \wedge q \Rightarrow p$
7.	$\sim P(y)$	Rule T,2, $p \wedge q \Rightarrow q$
8.	$P(y) \vee A(y)$	Rule T,4,6, Modus phones
9.	$A(y)$	Rule T,7,8, disjunctive syllogism
10.	$H(y)$	Rule T,5,9, Modus phones

5a) Prove by Mathematical Induction that for all $n \geq 1$,
$1 + 3 + 5 + ... + (2n - 1) = n^2$

Solution

Let the given statement P(n) be defined as $P(n) : 1 + 3 + 5 + ... + (2n - 1) = n^2$, for $n \in N$.
Note that P(1) is true
Since $P(1) : 1 = 1^2$
Assume that P(k) is true for some $k \in N$
i.e., $P(k) : 1 + 3 + 5 + ... + (2k - 1) = k^2$
Now, to prove that P(k + 1) is true
We have $1 + 3 + 5 + ... + (2k - 1) + (2k + 1)$
$= k^2 + (2k + 1)$
$= k^2 + 2k + 1$
$= (k + 1)^2$
Thus, P(k + 1) is true, whenever P(k) is true.
Hence, by the Principle of Mathematical Induction, P(n) is true for all $n \geq 1$

5b) Use the Euclidean algorithm to find gcd (414,662)

Solution:

Divide 662 by 414 to get a quotient of 1 and a remainder of 248.

$662 = 414 \times 1 + 248$

Divide 414 by 248 to get a quotient of 1 and a remainder of 166.

$414 = 248 \times 1 + 166$

Divide 248 by 166 to get a quotient of 1 and a remainder of 82.

$248 = 166 \times 1 + 82$

Divide 166 by 82 to get a quotient of 2 and a remainder of 0.

$166 = 82 \times 2 + 0$

Since the remainder is 0, the last non-zero remainder is the greatest common divisor.

Hence, the greatest common divisor of 414 and 662 is 2, which is
$gcd(414, 662) = 2$

6a) Use the chinese remainder theorem to solve the system of congruences
$X \equiv 2 \pmod 3$, $X \equiv 3 \pmod 4$, $X \equiv 2 \pmod 5$

The moduli are $m_1 = 3$, $m_2 = 4$, and $m_3 = 5$. Since $\gcd(3,4) = 1$, $\gcd(3,5) = 1$, and $\gcd(4,5) = 1$, the moduli are pairwise coprime, and a unique solution exists modulo $m_1 m_2 m_3$.

The product of the moduli is $M = 3 \times 4 \times 5 = 60$.

Next, we calculate $M_i = M/m_i$:

- $M_1 = \frac{60}{3} = 20$
- $M_2 = \frac{60}{4} = 15$
- $M_3 = \frac{60}{5} = 12$

We need to solve for y_i in the congruences $M_i y_i \equiv 1 \pmod{m_i}$:

- For $m_1 = 3$: $20y_1 \equiv 1 \pmod 3 \Longrightarrow 2y_1 \equiv 1 \pmod 3$. Multiplying by 2, we get $4y_1 \equiv 2 \pmod 3 \Longrightarrow y_1 \equiv 2 \pmod 3$. So, $y_1 = 2$.
- For $m_2 = 4$: $15y_2 \equiv 1 \pmod 4 \Longrightarrow 3y_2 \equiv 1 \pmod 4$. Multiplying by 3, we get $9y_2 \equiv 3 \pmod 4 \Longrightarrow y_2 \equiv 3 \pmod 4$. So, $y_2 = 3$.
- For $m_3 = 5$: $12y_3 \equiv 1 \pmod 5 \Longrightarrow 2y_3 \equiv 1 \pmod 5$. Multiplying by 3, we get $6y_3 \equiv 3 \pmod 5 \Longrightarrow y_3 \equiv 3 \pmod 5$. So, $y_3 = 3$.

The general solution is given by $x = a_1 M_1 y_1 + a_2 M_2 y_2 + a_3 M_3 y_3$. The values are $a_1 = 2, a_2 = 3, a_3 = 2$.

$$x = (2)(20)(2) + (3)(15)(3) + (2)(12)(3)$$

$$x = 80 + 135 + 72$$

$$x = 287$$

The unique solution modulo $M = 60$ is found by taking $x \pmod{60}$:

$$x \equiv 287 \pmod{60}$$

$$x \equiv 4 \cdot 60 + 47 \pmod{60}$$

$$x \equiv 47 \pmod{60}$$

6b) State fermat's little theorem and use it to find the remainder when 7^{222} is divisible by 13

Solution

Fermat's Little Theorem states that "if p is a prime number and a is an integer such that a is not divisible by p, then $a^{p-1} \equiv 1 (\text{mod} p)$."
This means that when a^{p-1} is divided by p, the remainder is 1.
This also can be written as $a^p \equiv a \pmod p$.

Since 13 is prime and $\gcd(7, 13) = 1$, Fermat's Little Theorem gives:

$$7^{12} \equiv 1 \pmod{13}.$$

Now reduce the exponent modulo 12:

$$222 \mod 12 = 6.$$

Thus,

$$7^{222} \equiv 7^6 \pmod{13}.$$

Compute powers of 7 mod 13:

- $7^2 = 49 \equiv 10 \pmod{13}$
- $7^3 = 7 \cdot 10 = 70 \equiv 5 \pmod{13}$
- $7^4 = 7 \cdot 5 = 35 \equiv 9 \pmod{13}$
- $7^5 = 7 \cdot 9 = 63 \equiv 11 \pmod{13}$
- $7^6 = 7 \cdot 11 = 77 \equiv 12 \pmod{13}$

7a) Solve the recurrence relation an =3 an−1 +2an−2 ; a0=1, a1=2 using characteristic roots

Solution:

The given recurrence relation is $a_n = 3a_{n-1} + 2a_{n-2}$, which can be rewritten as $a_n - 3a_{n-1} - 2a_{n-2} = 0$. The characteristic equation is found by substituting $a_n = r^n$, $a_{n-1} = r^{n-1}$, and $a_{n-2} = r^{n-2}$, which simplifies to:

$$r^2 - 3r - 2 = 0$$

$$r = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2(1)} = \frac{3 \pm \sqrt{9+8}}{2} = \frac{3 \pm \sqrt{17}}{2}$$

The two distinct characteristic roots are $r_1 = \dfrac{3+\sqrt{17}}{2}$ and $r_2 = \dfrac{3-\sqrt{17}}{2}$.

$$a_n = C_1\left(\frac{3+\sqrt{17}}{2}\right)^n + C_2\left(\frac{3-\sqrt{17}}{2}\right)^n$$

The initial conditions are $a_0 = 1$ and $a_1 = 2$.
For $n = 0$:

$$a_0 = C_1\left(\frac{3+\sqrt{17}}{2}\right)^0 + C_2\left(\frac{3-\sqrt{17}}{2}\right)^0 = C_1 + C_2 = 1 \quad \text{(Eq. 1)}$$

For $n = 1$:

$$a_1 = C_1\left(\frac{3+\sqrt{17}}{2}\right)^1 + C_2\left(\frac{3-\sqrt{17}}{2}\right)^1 = 2 \quad \text{(Eq. 2)}$$

From Eq. 1, $C_2 = 1 - C_1$. Substituting this into Eq. 2:

$$C_1\left(\frac{3+\sqrt{17}}{2}\right) + (1 - C_1)\left(\frac{3-\sqrt{17}}{2}\right) = 2$$

Multiplying by 2:

$$C_1(3+\sqrt{17}) + (1 - C_1)(3 - \sqrt{17}) = 4$$

$$3C_1 + C_1\sqrt{17} + 3 - \sqrt{17} - 3C_1 + C_1\sqrt{17} = 4$$

$$2C_1\sqrt{17} = 1 + \sqrt{17}$$

$$C_1 = \frac{1+\sqrt{17}}{2\sqrt{17}} = \frac{\sqrt{17}+17}{34}$$

Now, finding C_2:

$$C_2 = 1 - C_1 = 1 - \frac{1+\sqrt{17}}{2\sqrt{17}} = \frac{2\sqrt{17} - (1+\sqrt{17})}{2\sqrt{17}} = \frac{\sqrt{17}-1}{2\sqrt{17}} = \frac{17-\sqrt{17}}{34}$$

The solution to the recurrence relation is:

$$a_n = \left(\frac{17+\sqrt{17}}{34}\right)\left(\frac{3+\sqrt{17}}{2}\right)^n + \left(\frac{17-\sqrt{17}}{34}\right)\left(\frac{3-\sqrt{17}}{2}\right)^n$$

7b) Solve the recurrence relation by substitution $a_n = a_{n-1} + n$, $n \geq 1$, where $a_0 = 2$

Solution: Given $a_n = a_{n-1} + n$

$a_1 = a_0 + 1$

$a_2 = a_1 + 2 = a_0 + 1 + 2$

$a_3 = a_2 + 3 = a_0 + 1 + 2 + 3$

$a_n = a_0 + (1 + 2 + ... + n)$

$a_n = a_0 + n(n+1)/2$

Using $a_0 = 2$, we have $a_n = 2 + n(n+1)/2$

8a) Solve the recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = 0$ by generating functions.

Let the generating function for the sequence $\{a_n\}$ be $A(x) = \sum_{n=0}^{\infty} a_n x^n$.

The given recurrence relation is $a_n - 7a_{n-1} + 10a_{n-2} = 0$ for $n \geq 2$. We multiply this equation by x^n and sum from $n = 2$ to ∞:

$$\sum_{n=2}^{\infty} a_n x^n - 7\sum_{n=2}^{\infty} a_{n-1}x^n + 10\sum_{n=2}^{\infty} a_{n-2}x^n = 0$$

- The first sum is $\sum_{n=2}^{\infty} a_n x^n = \left(\sum_{n=0}^{\infty} a_n x^n\right) - a_0 - a_1 x = A(x) - a_0 - a_1 x.$
- The second sum is $$\sum_{n=2}^{\infty} a_{n-1}x^n = x\sum_{n=2}^{\infty} a_{n-1}x^{n-1} = x\sum_{k=1}^{\infty} a_k x^k = x\left(\left(\sum_{k=0}^{\infty} a_k x^k\right) - a_0\right) = x(A(x) - a_0)$$
- The third sum is $\sum_{n=2}^{\infty} a_{n-2}x^n = x^2\sum_{n=2}^{\infty} a_{n-2}x^{n-2} = x^2\sum_{k=0}^{\infty} a_k x^k = x^2 A(x).$

Substituting these back into the equation from Step 1 gives:

$$(A(x) - a_0 - a_1 x) - 7x(A(x) - a_0) + 10x^2 A(x) = 0$$

$$\boxed{A(x) = \frac{a_0 + (a_1 - 7a_0)x}{1 - 7x + 10x^2}.}$$

Factor the denominator:

$$1 - 7x + 10x^2 = (1 - 5x)(1 - 2x).$$

Thus

$$A(x) = \frac{a_0 + (a_1 - 7a_0)x}{(1 - 5x)(1 - 2x)}.$$

$$A(x) = \frac{C}{1 - 5x} + \frac{D}{1 - 2x}.$$

Solve for C and D:

$$a_n = C \cdot 5^n + D \cdot 2^n.$$

$$a_0: \quad C + D = a_0,$$

$$a_1: \quad 5C + 2D = a_1.$$

Solve:

$$C = \frac{a_1 - 2a_0}{3}, \qquad D = \frac{5a_0 - a_1}{3}.$$

8b) A coin is tossed 10 times. How many possible outcomes are there

Solution:

Each coin toss has 2 possible outcomes: Heads (H) or Tails (T).

For 10 independent tosses, the total number of possible sequences is: $2^{10}=1024$

9a) What is Graph isomorphism? Discuss the Graph isomorphism with examples

Two graphs G and G′ are isomorphic if there is a function f : V (G) →V (G′), from the vertices of G to the vertices of G′ such that

(i) f is one-one,

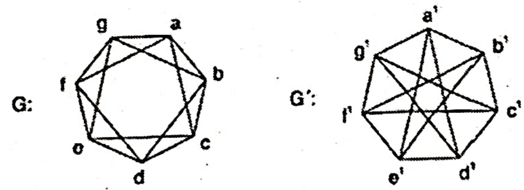
(ii) f is onto, and

(iii) For each pair of vertices u and v of G, {u, v}∈ E (G) if and only if { f (u), f (v)}∈ E(G′) (i.e., f-preserves adjacency).

If f : G → G′ is an isomorphism, then G and G′ are said to be isomorphic and if two graphs G and G′ are isomorphic then there may be several isomorphisms from G to G′.

Note: Two isomorphic graphs have the same number of vertices and same number of edges.

Example: Show that the two graphs shown in fig are isomorphic.



Solution: Define a mapping f: G → G′ such that

f (a) → a^1

f (b) → c^1

f (c) → b^1

f (d) → f^1

f(e) → c^1

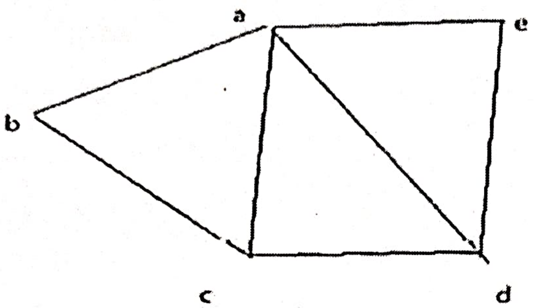
f (f) → g^1

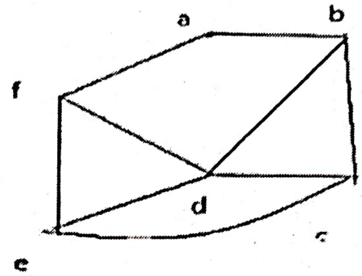
f (g) → d^1

clearly the mapping f is one-to-one and onto. f preserves the adjacency.

f G → G′ is an isomorphism note that both G and G′ have 7 vertices and 14 edges each. Every vertex in G and G′ is of degree 4.

9b) (i) What is chromatic number of a graph? Find the chromatic number pf the following graphs





Solution:
The chromatic number of a graph is the minimum number of colors needed to color the vertices of the graph such that no two adjacent vertices share the same color.

First Graph

The first graph contains the vertices a, b, c, d, and e. The vertices a, c, and d form a triangle (a 3-cycle), which is a clique of size 3. A graph containing a clique of size k requires at least k colors. Therefore, this graph requires at least 3 colors.
Color vertex **a** with Color 1. Color vertex **c** with Color 2. Color vertex **d** with Color 3.
Vertex **b** is only connected to **a** and **c**, so it can be colored with Color 3.
Vertex **e** is only connected to **a** and **d**, so it can be colored with Color 2.
Since the graph can be colored with 3 colors, and we know it requires at least 3, the chromatic number is 3.

Second Graph

The second graph contains the vertices a, b, c, d, e, and f. The vertices a, b, and d form a triangle (a 3-cycle), which is a clique of size 3. A graph containing a clique of size k requires at least k colors. Therefore, this graph requires at least 3 colors.
Color vertex **a** with Color 1. Color vertex **b** with Color 2. Color vertex **d** with Color 3.
Vertex **f** is only connected to **a** and **d**, so it can be colored with Color 2.
Vertex **e** is only connected to **d** and **f**, so it can be colored with Color 1.
Vertex **c** is only connected to **b,e** and **d**, so it can be colored with Color 4.
Since the graph can be colored with 3 colors, and we know it requires at least 3, the chromatic number is 4.

9 b) (ii) Elaborate the Euler and Hamiltonian graphs

Solution: Eulerian graphs have a circuit that traverses every edge exactly once, which is possible if and only if every vertex has an even degree. Hamiltonian graphs have a cycle that visits every vertex exactly once. The key difference is that Eulerian graphs are concerned with edges, while Hamiltonian graphs are concerned with vertices.

Feature	Eulerian Graph	Hamiltonian Graph
Focus	Traversing every **edge** exactly once	Visiting every **vertex** exactly once
Checkable Condition	Easy: All vertices must have an even degree for a circuit.	Difficult: NP-complete problem, no simple condition exists.

10a) Explain the graph traversals (BFS and DFS) with an example

Graph traversal algorithms systematically visit all nodes and edges in a graph. Breadth-First Search (BFS) and Depth-First Search (DFS) are two fundamental methods for this.

1. Breadth-First Search (BFS)

BFS explores the graph level by level. It starts at a source node, then visits all its immediate neighbors, then all their unvisited neighbors (nodes at distance 2 from the source), and so on. It utilizes a queue data structure to manage the order of visiting nodes.

Example: Consider a graph with nodes A, B, C, D, E and edges (A,B), (A,C), (B,D), (C,E). Starting from A:

- **Enqueue A.** Visited: {A}. Queue: [A]
- **Dequeue A.** Enqueue neighbors B, C. Visited: {A, B, C}. Queue: [B, C]
- **Dequeue B.** Enqueue neighbor D. Visited: {A, B, C, D}. Queue: [C, D]
- **Dequeue C.** Enqueue neighbor E. Visited: {A, B, C, D, E}. Queue: [D, E]
- **Dequeue D.** No unvisited neighbors. Queue: [E]
- **Dequeue E.** No unvisited neighbors. Queue: []

Traversal order: A, B, C, D, E.

2. Depth-First Search (DFS)

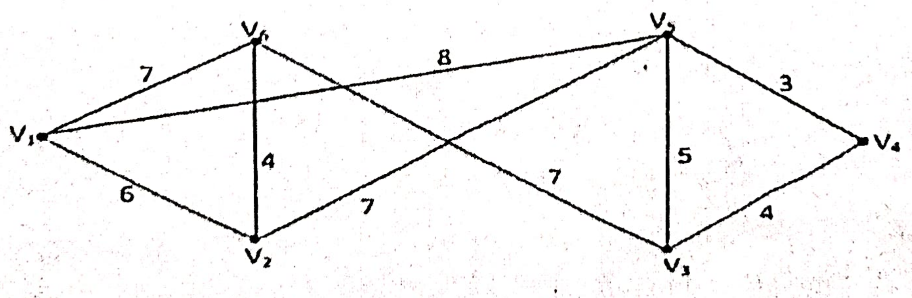
DFS explores as deeply as possible along each branch before backtracking. It starts at a source node, then explores one of its unvisited neighbors, then one of that node's unvisited neighbors, and so on, until it reaches a node with no unvisited neighbors. Then, it backtracks to explore other branches. It typically uses a stack or recursion (which implicitly uses the call stack).

Example: Using the same graph as above (A, B, C, D, E and edges (A,B), (A,C), (B,D), (C,E)). Starting from A:

- **Visit A.** Mark A as visited. Push A onto stack. Stack: [A]
- **Pop A.** Visit B (neighbor of A). Mark B as visited. Push B onto stack. Stack: [B]
- **Pop B.** Visit D (neighbor of B). Mark D as visited. Push D onto stack. Stack: [D]
- **Pop D.** No unvisited neighbors of D. Stack: []
- **Backtrack.** Return to A. Visit C (another neighbor of A). Mark C as visited. Push C onto stack. Stack: [C]
- **Pop C.** Visit E (neighbor of C). Mark E as visited. Push E onto stack. Stack: [E]
- **Pop E.** No unvisited neighbors of E. Stack: []
- **Backtrack.** Return to A. No unvisited neighbors of A. Stack: []

Traversal order (one possible path): A, B, D, C, E. (Order can vary based on neighbor selection).

10b) Use Prim's algorithm to find a minimal spanning tree for the following weighted graph



Solution:

Step 1: Initialize the tree

Start with an arbitrary vertex, for example, V_1. The set of vertices in the tree is $\{V_1\}$. The edges connected to V_1 are (V_1, V_6) with weight 7, (V_1, V_2) with weight 6, and (V_1, V_5) with weight 8.

Step 2: Add the next minimum edge

The minimum weight edge is (V_1, V_2) with weight 6. Add V_2 to the tree. The set of vertices in the tree is $\{V_1, V_2\}$. New edges to consider: (V_2, V_6) with weight 4, (V_2, V_5) with weight 7, and (V_2, V_3) with weight 7.

Step 3: Add the next minimum edge

The minimum weight edge among those connected to $\{V_1, V_2\}$ is (V_2, V_6) with weight 4. Add V_6 to the tree. The set of vertices in the tree is $\{V_1, V_2, V_6\}$. New edges to consider: (V_6, V_5) with weight 7.

Step 4: Add the next minimum edge

The minimum weight edge connected to $\{V_1, V_2, V_6\}$ is (V_5, V_3) with weight 5 (connected via V_5, which is connected via V_2 or V_6 with weight 7, or via V_1 with weight 8). The edge (V_2, V_5) has weight 7, the edge (V_6, V_5) has weight 7. The edge (V_5, V_3) has weight 5. Add V_5 to the tree. The set of vertices in the tree is $\{V_1, V_2, V_6, V_5\}$. New edges to consider: (V_5, V_3) with weight 5 and (V_5, V_4) with weight 3.

Step 5: Add the next minimum edge

The minimum weight edge connected to $\{V_1, V_2, V_6, V_5\}$ is (V_5, V_4) with weight 3. Add V_4 to the tree. The set of vertices in the tree is $\{V_1, V_2, V_6, V_5, V_4\}$. New edges to consider: (V_4, V_3) with weight 4.

Step 6: Add the final edge

The minimum weight edge connected to $\{V_1, V_2, V_6, V_5, V_4\}$ is (V_4, V_3) with weight 4. Add V_3 to the tree. The set of vertices in the tree is $\{V_1, V_2, V_6, V_5, V_4, V_3\}$. All vertices are in the tree.

Answer:

The edges in the minimal spanning tree are (V_1, V_2), (V_2, V_6), (V_5, V_4), (V_4, V_3), and (V_2, V_5) or (V_6, V_5) or (V_5, V_3). The total weight of the minimal spanning tree is $6 + 4 + 3 + 4 + 7 = 24$ (using edge (V_2, V_5)).

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